HORNBEAK

Laboratory Methods of Measuring Self-Inductance

Physics

A. M.

1909

LABORATORY METHODS OF MEASURING SELF-INDUCTANCE

BY

JOHN WESLEY HORNBEAK

B. S. Illinois Wesleyan University, 1906

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF ARTS

IN PHYSICS

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1909

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 15, 1909

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

JOHN WESLEY HORNBEAK

ENTITLED LABORATORY METHODS FOR MEASURING SELF-INDUCTANCE

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Arts in Physics

Chas. T. Knipp
In Charge of Major Work

A. P. Carman
Head of Department

Recommendation concurred in:

Committee
on
Final Examination

TABLE OF CONTENTS

LABORATORY METHODS OF MEASURING SELF-INDUCTANCE

INTRODUCTORY

Preliminary Remarks.- In doing this work the primary object was not to compare the methods, but to determine for each method the working conditions under which the best results may be expected with the d'Arsonval type of galvanometer. Consequently, as an examination of the data tables will show, the agreement of results, in some cases, is not as good as is possible with the apparatus used,- the reason being that the values of certain quantities were varied simply to find out whether or not the change would prove to be an advantage.

Development of Equations which define L.- An electric current flowing in a circuit is equivalent to a magnetic shell, and has linked thru it a definite number of lines of magnetic induction. If the permeability of the surrounding region does not vary, the field strength H at any given point is proportional to the current, and hence the value of the total flux is also proportional to the current. In symbols, we may write it $N = Li$ ... .(1). The proportionality factor, L, is called the Coefficient of Self-Induction, or the Inductance of the circuit. Its value depends only upon the geometrical relations of the circuit and the magnetic permeability of the surrounding medium. The equa-

tion defines L as the total number of lines of magnetic force threading the circuit when it is traversed by unit current.

From the general law[1] of induced electro-motive force, established experimentally by Faraday, we have

$$e = - dN/dt, \quad \ldots\ldots(2).$$

where e is the instantaneous value of the electromotive force of self-induction, and dN/dt is the rate of change of the magnetic flux. The minus sign gets its interpretation from Lenz's law and means that the induced E.M.F. is directed so as to oppose the variation in N.

Substituting in (2) the value of N from (1), and assuming L constant, we get

$$e = -d(Li)/dt = -Ldi/dt,$$

and $$L = -e/di/dt \quad \ldots\ldots(3).$$

This equation is seen to define L as the ratio of the E.M.F. of self-induction to the rate of change of the current. If e is unity when di/dt is unity, L is unity also. Hence, in practical units, the self-induction of a coil is one henry if a variation in the current at the rate of one ampere per second, induces an opposing E.M.F. of one volt.

APPARATUS

For the sake of convenience in reference, and to avoid otherwise necessary repetition, a general description of the apparatus used for the different methods will be included in this paragraph.

Resistance.- Resistance boxes were used for all non- induct-

[1] Foster & Porter, Elect. & Mag., p.p. 364, 365, 2nd Ed. 1903.

ive resistances. These were standards, accurate to 1/50 of 1%, made by Hartmann and Braun, Germany.

Galvanometers.- No's 2717A and 2717C are the Type - H instrument of The Leeds and Northup Co., Philadelphia. They have a telescope and semi-circular scale. These galvanometers are constructed so as to be applicable for either ballistic or aperiodic work.

No's 2002 and 2183 were made by Nalder Bros. of London. They are the kind that require a lamp and scale. To get the spot of light, however, the image of the filament of an ordinary electric light bulb was focused upon the scale.

No's 2557B, 2557C and 2557D, are the Ayrton-Mather galvanometers made by Robt. W. Paul, London. They have a suspended system which is removable, and this removable system determines whether the galvanometer is aperiodic or ballistic. The coil of each is imbedded in a very light cylinder of non-magnetic material, and the controlling helix below the coil is extremely delicate.

The following table of constants gives a general idea of the galvanometers.

Table I

Galv.	Resist. in ohms	Scale dist. cm.	T seconds	λ
2717A	491	51.1	5.68	.0232
2717C	526	50.8	5.52	.0221
2183	45.7	111.0	4.63	.0172
2002	706.0	96.3	16.08	.1070
2557B	326.5	110.0	4.5	.0103
2557C	1160	108	4.93	.0111
2557D	1183	107	4.70	.0122

Condensers.- Standard mica condensers, made by the Leed's & Northup Co., were used. In some instances, however, at times when a sufficient number of standards were not available, it was necessary to use other condensers. Where this was done, the value of the capacity in the data column is checked with red ink. These last mentioned are mica condensers, but their values are not known to a degree of accuracy within one-half of one per cent.

Slide-wire Bridges.- The bridges used, which were constructed in 1905 in the department shop, have manganin slide-wires,- length, one meter; resistance, .64 ohm.

Inductance Coils.- Coils No. 132 and 133 are Hartmann & Braun standards, .001henry and .1 henry, respectively. The exact certificated value from the Reichsanstalt for the former is .000998 ± .000002; and for the latter, .0997± .0002.

As the rest are not standards, their values will not be given here. No's 1520, 1523 and 1527 are wound on spools of wood, the grooves of which are 2.7 cm. wide; and the mean radii of the coils are about 3 cm.

No's 330, 460, 590, 660 and 790 are coils which I wound in the shop of the physics department. The spools were turned of maple and all made alike;- grooves rectangular, 3 cm. wide; radius of cores, 2 1/2 cm. Copper wire #22 BS gauge was used. The number of the coil indicates in each case the number of turns. The spools were boiled in paraffin for two hours before winding.

With the exception of the five coils just mentioned, all of the apparatus which was used, came from the general stock

in use by the advanced students in electrical measurements.

MAXWELL'S METHOD[1]

Theory and Manipulation.- Since three of the methods which were studied are modifications of Maxwell's method of measuring self-inductance, his method will be briefly discussed.

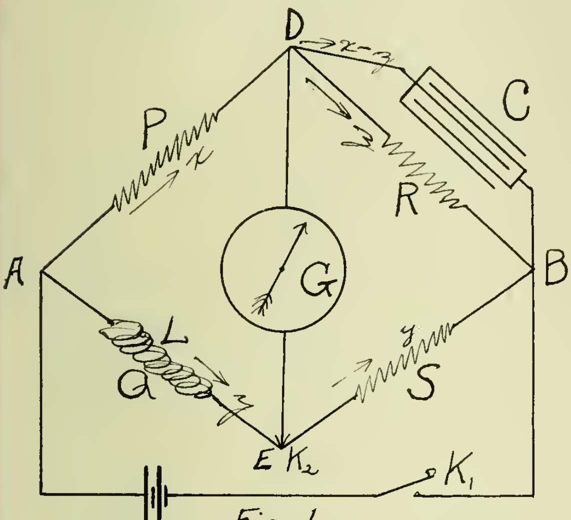

Fig. 1.

A coil of self-inductance L and ohmic resistance Q, is put into one branch of a Wheatstone's bridge, and into the other three branches are connected non-inductive resistances, P, R and S, as shown in the figure. Around R is shunted a capacity, C. By means of the sliding contact at E, the balance for steady currents is obtained.

The theory of the method rests upon the fact that the mathematical expression[2] for the instantaneous value of the P.D. between the terminals of a condenser when charging, is a function of the time exactly similar to that for the instantaneous effective E.M.F. in a circuit containing resistance and self-inductance. Consequently, the resistance and capacity may be so adjusted that when K_1 is closed after K_2, the retarding effect on the current in the branches Q and R, due to the coil and capacity, respectively, can be made to balance each other; so that the potential at D and E will be the same at every instant. When such adjustment is made, no current will flow thru

1. "Elect. & Mag.", Maxwell, Vol. II., p. 377.
2. "Alternating Currents", Bedell & Crehore, p.p. 49, 119 (Note.- Referring to formulas (21) and (168), put Q_o=0 in the latter, reduce both to E.M.F. equations, then compare them).

the galvanometer. We shall now determine the conditions under which the potential at D is always equal to that at E.

Let x and z be the quantities of electricity which have passed thru the branches AD and DB, respectively, at the time <u>t</u> after closing the battery circuit. Then x - z is the charge on the condenser at that instant. Since dz/dt is the value of the current, the P.D. between the plates of the condenser is, by Ohm's law, Rdz/dt, and we have

$$x - z = RCdz/dt \qquad (4).$$

where C is the capacity of the condenser.

Since there has been no current thru the galvanometer during the interval <u>t</u>, the same quantity has passed thru the two branches AE and EB. We shall call this quantity y. Observing that the potential drop is the same across DB and EB, we have

$$Sdy/dt = Rdz/dt \qquad (5).$$

Likewise, the potential drop across AD is equal to that across AE, and its value is Pdx/dt. However, the effective E.M.F. in the branch AE, given by Ohm's law, is equal to the algebraic sum of the total potential drop and the E.M.F. of self-induction. That is

$$Qdy/dt = Pdx/dt - Ld^2y/dt^2 \qquad (6).$$

Transposing in (4) and differentiating with respect to t,

$$dx/dt = dz/dt + RCd^2z/dt^2.$$

Substituting this value in (6), we have

$$Qdy/dt + Ld^2y/dt^2 = P(dz/dt + RCd^2z/dt^2) \qquad (7).$$

But $dy/dt = R/S \times dz/dt$, from equation (5).

Substituting in (7),

$$(QR/S)(dz/dt) + (RL/S)(d^2z/dt^2) = P(dz/dt + RCd^2z/dt^2). \qquad (8).$$

Integrating (8), we have

$$QRz/S + (RL/S)(dy/dt) = P(z + RCdz/dt);$$

and $$z(QR/S - P) = (PRC - RL/S)dz/dt \quad \text{.....................(9).}$$

Since the bridge is balanced for steady currents, QR = PS and the left-hand member of (9) is equal to zero. Hence it follows at once that L/S = PC

and $$L = PSC = QRC.$$

This is the codition of no current thru the galvanometer on making or breaking the battery circuit. If the resistances are in ohms and C in farads, L will be expressed in henrys.

Objections to the Method.- A double adjustment is necessary to obtain a balance. That is, if the bridge is first balanced for steady currents, this balance is destroyed when one of the resistances is changed to eliminate the ballistic throw due to the induction current when K is closed first. Hence a new adjustment for steady currents is necessary before another attempt to eliminate the throw is made; and so on. Consequently the final adjustment of resistances is a tedious process, and for this reason the method did not come into general use.

ANDERSON'S MODIFICATION OF MAXWELL'S METHOD

Theory and Manipulation.- In order to avoid the difficulty just mentioned, Anderson introduced a non- inductive resistance, r, into the galvanometer circuit and connected the condenser as shown in Fig. 2. This resistance can be adjusted until the time-constant of the condenser is equal to that of the coil, and the changes in the value of r do not affect the balance

Phil. Mag. (1891), Vol. XXXI., p. 334.
Phil. Mag. (1904), Vol. VII., p. 586.

for steady currents.

Supposing this adjustment of r to be made, no current will flow thru G when K_2 is closed before K_1. Let x, y and z represent the quantities which have passed thru the branches DB, EB and DN, respectively, at the time t after closing K_1. Then the quantities x + z and y have traversed AD and AE, respectively, and z is the charge on the condenser. Also, let C be the capacity of the condenser and L the self-inductance of the coil.

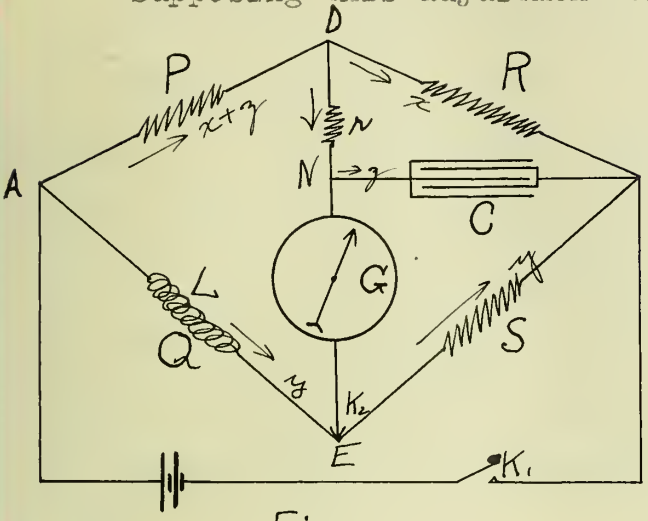

Fig. 2.

Since there is no potential difference at any instant between the points N and E, the potential drop from N to B is the same as that from E to B. That is

$$z/C = S\,dy/dt \qquad (10).$$

Similarly, since r and C are in parallel with R, the drop of potential from D to B is the same by the two paths, and

$$R\,dx/dt = r\,dz/dt + z/C \qquad (11).$$

The total P. D. from A to E is equal to the effective electromotive force plus the E. M. F. necessary to overcome self-induction. Expressing this in symbols and equating it to the P. D. from A thru D to N, we have

$$Q\,dy/dt + L\,d^2y/dt^2 = P\,dx/dt + P\,dz/dt + r\,dz/dt \qquad (12)$$

Obtaining from (10) and (11) the values of dy/dt, d^2y/dt^2 and dx/dt, and substituting in (12), we get

$$(Q/S - P/R)cz = (Pr/R + P + r - L/CS)\,dz/dt \qquad (13).$$

But since the bridge is balanced for steady currents, $P/R = Q/S$, and the left hand member of (13) is equal to zero.

Therefore, $L/CS = Pr/R + P + r$;

and $$L = C\{r(Q+S) + PS\}$$

This, then, is the condition for balance for varying currents. If $r = 0$, it reduces to Maxwell's formula.

<u>Data and Discussion</u>.- It was soon discovered that the chief source of error in this method is in the determination of r. Hence, most of the discussion will be confined to this quantity.

The coils were always packed with felt in a double-walled can which was lined with thick asbestos.

When a resistance box is used in each branch of the Wheatstone's bridge, the resistance of a part of a millimeter of the slide wire is neligible. Hence, the bridge readings were all recorded to the nearest whole millimeter.

The bridge was first balanced by increasing r, then by decreasing r; and if the two values were not the same, the mean was taken.

TABLE II.

Galv. 2557 B; coil 133; 2 dry cells.

No.	P in ohms	Q' in ohms	R ohms	S' ohms	Mean r	C m f	a mm.	L	t
(1)	29	29.30	200	204	510	.8	540	.1002	19.0°
(2)	29	29.37	1000	1020	67	1.0	433	.0999	19.3
(3)	29	29.41	150	153	522	1.0	455	.1000	19.7
(4)	29	29.54	500	510	204	.8	293	.1000	20.3
(5)	29	29.47	800	815	90	1.0	133	.0997	20.5
(6)	29	29.58	150	154	518	1.0	440	.0999	21.7
(7)	29	29.90	150	156	513	1.0	392	.1002	25.1
(8)	29	29.95	400	417	196	1.0	390	.0998	25.4
(9)	30	29.49	170	169	476	1.0	600	.0999	20.8
(10)	29	29.29	500	510	159	1.0	536	.1006	18.5
(11)	29	29.35	1000	1020	67	1.0	472	.0999	19.1
(12)	29	29.38	700	714	106	1.0	426	.0996	19.4
(13)	29	29.42	301	305	271	1.0	386	.0997	19.8
(14)	29	29.48	500	512	157	1.0	401	.1000	20.6
(15)	29	29.51	800	820	90	1.0	375	.1006	21.0
(16)	29	29.54	200	205	402	1.0	392	.1005	21.3

No determinations were discarded. In (1)-(9), Table II, the building was steady and the needle would come completely to rest. In (10)-(16), the needle was vibrating slightly-about .2 mm amplitude-and the effect is seen on the results.

Q' = the resistance of the coil corrected for temperature. An accurate thermometer, graduated to tenths of degrees, was used. S' is the resistance box value in the S-branch. Q and S are got

from these values by adding the resistance of the proper length of the bridge wire (resistance = .00064 ohms per m.m.) The values in the column headed a, are the readings on the bridge-wire, of the sliding contact E (See Fig. 2) when the balance was obtained.

The Carey Foster method was used in determining the resistance of the coil. Five determinations were made, as follows:

TABLE III.

Temperature.	Resistance.
21.40°	29.546 ohms.
21.43	29.553
21.50	29.555
21.52	29.559
21.56	29.563

It was decided to call the resistance 29.56 ohms, at 21.5°, and 29.55 ohms at 21.4°. The correction made in getting Q' was .01 ohm per .1 degree.

TABLE IV.

Galv. 2557 C; coil 1520; 2 dry cells.

No.	P	Q'	R	S'	a m.m.	C	r mean	L	t
(1)	6	6.06	200	212	500	.7	35	.00625	23.9°
(2)	6	6.11	201	210	315	.1	283	.00626	26.0
(3)	6	6.12	201	210	300	.5	52	.00627	26.6
(4)	6	6.14	147	156	428	.7	49.5	.00630	27.2
(5)	6	6.11	147	156	451	.4	90	.00624	26.2
(6)	6	6.11	147	156	460	.2	187	.00627	26.0

None discarded. Some difficulty due to vibrations. In fact,

when using the Ayrton & Mather galvanometer, there was almost continual trouble because of vibrations. Hereafter, to indicate whether or not this difficulty was met, we shall say: "G steady" or "G unsteady".

Resistance of coil 1520 at 25.5° = 6.09 ohms.

TABLE V.

Galv. 2557C; coil 1527; 3 dry cells.

No.	r_1	r_2	r mean	Q'	C	S'	a mm.	L	t
(1)	34	35	34.5	12.87	.9	122	575	.00564	22.0°
(2)	30	30	30	12.87	1.0	122	571	.00565	22.2
(3)	47	47	47	12.87	.7	122	568	.00557	22.4
(4)	70	72	71	12.88	.5	122	565	.00561	22.7
(5)	150	153	151.5	12.88	.25	122	562	.00553	22.8

None discarded. P = 13, R = 100, Q' = 10 ohms series resistance plus resistance of coil corrected for temp. G unsteady; vibrating deflection, 1/2 mm. amplitude.

Resistance of coil = 2.89 ohms @ 24°.

TABLE VI

Galv. 2183; coil 133; 2 dry cells.

No.	P	Q'	R	S'	a mm.	r		C	L	t
(1)	30	29.74	91	90	110	114	115	6.07✓	.1002	23.3°
(2)	30	29.74	91	90	111	142	142	5.07✓	.1004	23.3
(3)	30	29.74	91	90	120	809	811	1.00	.1002	23.2
(4)	30	29.73	121	120	190	192	192	3.09✓	.1004	23.2
(5)	30	29.72	121	120	208	144	144	3.98✓	.1006	23.0

For meaning of checked values, see page 4.

None discarded

TABLE VII

Galv. 2183; coil 1523; 2 dry cells.

No.	P	Q"	Q'	R	S'	a mm.	r		C	L	t
(1)	20	15	19.86	120	120	306	14	14	1.0	.00438	22.5
(2)	20	15	19.86	120	120	307	45	46	.5	.00440	22.5
(3)	20	15	19.86	120	120	308	61	62	.4	.00442	22.4
(4)	15	10	14.86	100	100	299	82	82	.4	.00439	22.3
(5)	15	10	14.86	100	100	300	29.5	29.5	.9	.00442	22.3

None discarded. Q" = series resist., Q' = Q" + resist. of coil. The Nalder galvanometer is not so sensitive as the Ayrton & Mather, but it always came to rest. Hence the agreement of results in this table is better than was got for glav. 2557C, coil 1527. It is impossible to make accurate measurements with a zero method if the spot of light on the scale is vibrating perceptably.

TABLE VIII

Galv. 2183; coil 1520; 2 dry cells.

No.	P	Q'	R	S'	a mm.	r		C	L	t
(1)	29.1	29.63	90	92	353	113	114	6.07	.1005	22.2°
(2)	29.1	29.63	90	92	360	140	141	5.07	.1007	22.2
(3)	29.1	29.62	137	140	298	92	93	5.07	.1006	22.1
(4)	29.1	29.62	137	140	302	124	124	3.98	.1003	22.1
(5)	29.1	29.61	137	140	306	166	167	3.09	.1002	22.0

None discarded. The results seem to indicate that the nominal values of the capacities are a little high. No series resist. Q' = resist. of coil, cor. for temp.

TABLE X

Galv. 2002; coil 1520; 3 dry cells.

(1)	16	10	15.95	70	70	268	60	60	1.	.00632	18.8°
(2)	16	10	15.95	100	100	220	40	40	1.	.00627	19.0
(3)	16	10	15.95	100	100	217	121	121	2.09	.00628	19.1
(4)	21	15	20.95	110	110	231	5.5	5.5	2.09	.00636	19.2
(5)	21	15	20.96	90	90	252	54	54	.8	.00634	19.4

None discarded. It should be noticed that in this table and all preceding, the Q of the formula is equal to Q' + a in ohms. That is, Q = Q' + .00064a; and S = S' +(1000 - a)(.00064).

Up to this time, as has been suggested, the value of Q was built up in the following way. First, the resistance of the coil was determined by computing and applying the necessary increment (using the temperature coefficient of copper wire as .00388); then the resistance of the length a of the bridge wire was added; and also the series resistance, if any. This is a much longer process than getting Q from the simple bridge formula: Q = PS/R. So a number of values of Q from the foregoing data were determined by both methods and compared, and it was found that the former method is no more accurate than the latter. Hence, the bridge formula was used from this time on; and temperature readings were no longer taken.

In the remaining tables, Q' = the series resistance, and S' = the resistance-box value in the S-branch of the bridge.

TABLE XI

Galv. 2557 C; coil 660; 2 dry cells.

No.	P	Q'	Q	R	S'	S	r		a mm.	C	t
(1)	20	12	20.05	100	100	100.26	183	183	598	1.0	.0240
(2)	20	12	20.05	100	100	100.25	315	317	615	.6	.0240
(3)	20	12	20.03	200	200	200.26	91	91	588	1.0	.0240
(4)	30	22	30.04	200	200	200.25	104	104	613	.8	.0240
(5)	30	22	30.05	150	150	150.23	122	123	636	.9	.0239

The resistance of this coil is 7.67 ohms @ 22.2°. G was steady and the effect is seen in the agreement of results. No determinations were discarded.

TABLE XII

Galv. 2557 C; coil 330; 3 dry cells.

No.	P	Q'	Q	R	S'	S	r		a mm.	C	L
(1)	20	16	19.73	203	200	200.28	7.1	7.1	568	1.	.00556
(2)	20	16	19.73	203	200	200.27	17	18	571	.7	.00550
(3)	20	16	19.75	304	300	300.27	16	16	580	.5	.00556
(4)	14	10	13.82	304	300	300.20	31	32	694	.4	.00560
(5)	14	10	13.76	102	100	100.26	43	48	594	.8	.00550

None discarded. G. not very steady.

TABLE XIII

Galv. 2557 C; coil 790; 2 dry cells.

No.	P	Q'	Q	R	S'	S	r		a mm.	C	C
(1)	20	10	19.42	206	200	200.30	128.5	128.5	534	1.	.0322
(2)	20	10	19.44	206	200	200.29	166	166	550	.8	.0324
(3)	20	10	19.45	206	200	200.28	145	145	558	.9	.0323
(4)	25	16	25.38	150	152	152.30	181	181	532	.9	.0324
(5)	25	16	25.38	150	152	152.30	282	284	534	.6	.0324

None discarded. G. steady. The method is seen to be just as sensitive to changes in an "r" of 100-200 ohms as when r is below 50 ohms.

Resistance of coil - 9.09 ohms @ 22.8°.

TABLE XIV

Galv. 2557 D; coil 460; 2 dry cells.

No.	P	Q'	Q	R	S'	S	r	a mm.	C	L
(1)	20	15	20.06	110	110	110.34	63 | 63	472	1.0	.01042
(2)	20	15	20.06	110	110	110.34	97 | 99	470	.7	.01049
(3)	20	15	20.04	200	200	200.37	76 | 77	425	.5	.01043
(4)	30	25	30.05	200	200	200.36	24 | 25	430	.9	.01048
(5)	30	25	30.06	200	200	200.37	36 | 38	425	.4	.01042

None discarded. Comparing No. (4) with the rest, we have another illustration of the fact that the method is no more sensitive to locating the value of so small an r, than for r somewhat larger. Therefore, the percent of error is greater, on the average. Former tables show the same thing.

Resistance of coil = 4.76 ohms @ 22°.

TABLE XV

Glav. 2717 c; coil 590; 2 dry cells.

No.	P	Q'	Q	R	S'	S	r	a mm.	C	L
(1)	20	13	19.87	101	100	100.34	136 | 138	476	1.0	.0185
(2)	20	13	19.87	101	100	100.34	176 | 176	476	.8	.0185
(3)	30	23	30.02	200	200	200.17	89 | 89	728	.7	.0185
(4)	30	23	30.02	200	200	200.17	54 | 54	728	1.0	.0184
(5)	24	17	24.02	300	300	300.19	91 | 93	705	.5	.0185

TABLE XVI

Galv. 2717 C; coil 1523; 3 dry cells.

No.	P	Q'	Q	R	S'	S	r		a mm.	C	L
(1)	20.1	15	20.17	100	100	100.37	55	55	420	.5	.00432
(2)	20.1	15	20.17	100	100	100.36	73	75	439	.4	.00437
(3)	20.1	15	20.19	80	80	80.34	28	28	466	1.0	.00442
(4)	20.1	15	20.19	80	80	80.34	70	70	470	.5	.00433
(5)	30.0	25	30.11	80	80	80.29	28	28	560	.8	.00440
(6)	30.1	25	30.15	200	200	200.34	12	12	461	.5	.00440

None discarded. The balancing process was much more difficult with this galvanometer than with the ones used previously, on account of a slight parallax effect in reading the scale. Other telescopes were tried but the trouble could not be got rid of entirely.

Summary of Conclusions.- 1. The bridge should first be balanced by increasing r, then by decreasing r, and the mean taken. The very fact that the two values of r, thus obtained, are not always the same is sufficient reason why this should be done.

2. Since the resistance r is in the galvanometer circuit, it obviously should not be large, on account of diminishing the sensitiveness of the galvanometer. On the other hand, the data shows that the arrangement is no more sensitive to changes in r when its value is only a few ohms, than when it is a hundred ohms, or more. Therefore r should not be too small, because the percent of error, due to a mistake in the value of r, increases as r decreases. In general, a value of r between

50 and 200 ohms will give the best results.

3. The value of Q should be computed from the bridge formula. (See page 14).

4. It is better to use a series resistance with coils of small resistance. By thus making Q larger, any mistake in getting its value becomes a smaller percent of error.

5. The Ayrton-Mather galvanometer gives much the best results when the needle will come to rest; but when there is any trouble at all due to vibrations, the Nalder is just as good. This simply means that in order to do accurate work with a galvanometer when using a zero method, the <u>needle must come entirely to rest</u>.

6. The Type-H requires a telescope and, for this reason, it was found more difficult to judge when the zero-point was reached with this galvanometer.

RIMINGTON'S MODIFICATION OF MAXWELL'S METHOD[1]

<u>Theory and Manipulation</u>.- In this method the apparatus is connected just as in Maxwell's, except that the condenser is shunted over only a part of R, Fig. 3. N is a point on the

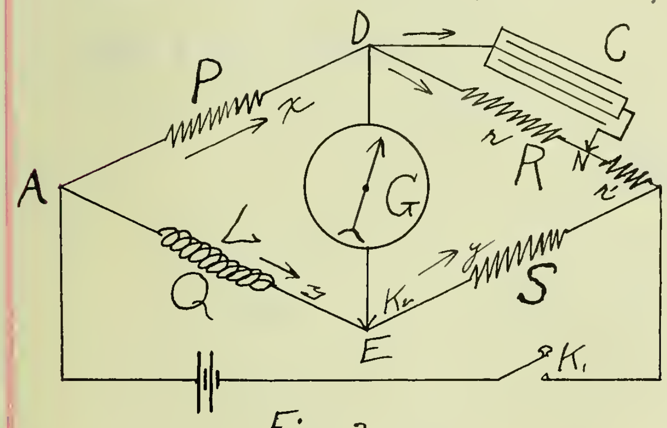

Fig. 3.

sliding contact between the two resistance boxes, r and r'; so that by transferring plugs from the one to the other, the resistance in parallel with the condenser can be varied without changing the total re-

[1] Phil. Mag. (1887). Vol. XXIV, p. 54.

sistance, R, in the branch DB.

Supposing the bridge is balanced for steady currents and that both keys are closed, let x and y represent the values of the current thru ADB and AEB, respectively, after the steady condition has been reached. Then if K is opened, the coil L will send a momentary current thru the galvanometer, the integral of which is the quantity[1]:

$$\frac{Ly}{\frac{G(R+S)}{G+R+S}+P+Q} \times \frac{R+S}{G+R+S} = \frac{LyR}{R(P+Q)+G(P+R)},$$

since PS = QR.

Also, the condenser will discharge a quantity thru the galvanometer which is expressed by

$$Cxr\frac{r}{\frac{G(P+Q)}{G+P+Q}+R+S} \times \frac{P+Q}{G+P+Q} = \frac{Cxr^2P}{R(P+Q)+G(P+R)},$$

since PS = QR.

But these quantities pass thru the galvanometer in opposite directions and, if there is no throw, they are equal.

Therefore $LyR = Cxr^2P$,

and $L = (CPr^2/R)(x/y)$..............................(14).

But $x/y = Q/p$;

Hence $L = CQr^2/R$..................................(15).

By making r = R, this becomes Maxwell's formula.

If the above relations are developed for the state of affairs when the battery circuit is made rather than broken, a result identical with (15) is obtained, and so the formula holds for both cases.

[1] Foster & Porter, Elect. and Mag., p. 384.

It should be noticed that in this discussion the condition that the potential at the two points D and E rises at the same rate, is not satisfied. Hence the assumption is made that the whole quantity of electricity, in both directions, passes thru the galvanometer before the needle has sensibly moved. Otherwise there will be a double kick, even tho' the time integral of the current thru the galvanometer is zero.

Data and Discussion.- In this method, as in Anderson's, the ~~only~~ chief source of error is in the determination of r.

For all the data: Q' = series resistance, S' = resistance box value in S-branch. C is in micro-farads and all resistances are in ohms.

TABLE XVII

Galv. 2557 B; coil 1523; 3 dry cells.

No.	P	Q'	Q	R	r mean	S'	a mm	C	L
(1)	15	10	14.94	120.8	74.1	120	65	6.42	.00436
(2)	15	10	14.94	120.8	80.7	120	70	5.42	.00437
(3)	15	10	14.96	200.8	104.7	200	90	5.42	.00442
(4)	35	30	34.94	200.8	62.5	200	55	6.42	.00436
(5)	35	30	34.93	100.8	45.1	100	42	6.22	.00438

None discarded. The wire of a second bridge was used between r and r', but moving the point N one-half the length of the wire could barely be detected in (2), (5). In (1), (3), (4) it could not be detected. G was unsteady.

TABLE XVIII

Galv. 2557 B; coil 133; 3 dry cells.

No.	P	Q'	Q	R	r mean	S'	a mm.	C	L
(1)	30	0	29.94	800.8	649.1	800.1	270	6.42	.1009
(2)	30	0	29.93	800.8	705.1	800.1	252	5.42	.1006
(3)	30	0	29.95	800.8	675.1	800.1	235	5.90	.1005
(4)	30	0	29.95	700.8	632	700.1	215	5.90	.1007
(5)	30	0	29.94	700.8	585	700.1	62	6.90	.1008

None discarded. Did not use sliding contact at N, on the second bridge-wire, as the arrangement was not sensitive to so small a change in r. Used three cells as the large R cut down the current. The results show a constant error which may be due to the capacities. (See page 4).

TABLE XIX

Galv. 2557 C; coil 1520; 2 dry cells.

No.	P	Q'	Q	R	r mean	S'	a mm.	C	L
(1)	22	16	22.24	130	75	131	307	6.6	.00634
(2)	20	14	20.08	120	75	120	76	6.6	.00621
(3)	20	14	20.10	120	77	120	25	6.4	.00636
(4)	16	10	16.31	120	83.5	122	455	6.6	..00626
(5)	16	10	16.31	120	85	122	215	6.4	.00628
(6)	20	14	20.10	100	70	100	215	6.4	.00630
(7)	20	14	20.10	100	67.5	100	228	6.9	.00632
(8)	20	14	20.10	100	90	100	290	3.85	.00627

Discarded three determinations which were off mors than 20%, due probably to a loose plug, tho' the trouble was not found.

For No. (3) the bridge-wire between r and r' was taken out because it was found to be of no advantage. It was bot put in again. For No's (1)-(4), G was not steady; for No's (5)-(9), G was steady; note the difference in agreement of results.

TABLE XX

Galv. 2557 D; coil 590; 2 dry cells.

No.	P	Q'	Q	R	r		S'	a mm.	C	L
(1)	20	13	20.02	250	215	215	250	597	5.	.0185
(2)	20	13	20.02	300	263	263	300	590	4.	.0185
(3)	20	13	20.02	350	283	284	350	575	4.	.0184
(4)	30	23	30.03	300	192	192	300	597	5.	.0185
(5)	30	23	30.04	200	165	165	200	613	4.5	.0184

None discarded. G steady.

Beginning at this time, the independent values of r got by balancing in opposite directions, were recorded, rather than the mean.

The agreement of the corresponding values of r in Table XX shows that no sensitiveness is lost by having r as large as 200 ohms.

TABLE XXI

Galv. 2717 C; coil 790; 3 dry cells.

No.	P	Q'	Q	R	r		S'	a mm.	C	L
(1)	30	21	30.32	500	374	375	505	371	3.85	.0327
(2)	30	21	30.33	500	433	434	505	366	2.85	.0324
(3)	30	21	30.40	400	334	334	405	482	3.85	.0326
(4)	30	21	30.52	300	281	281	305	663	4.04	.0324
(5)	30	21	30.49	200	190½	190½	203	610	5.89	.0325

None discarded. Note again the agreement in values of r, show-

ing that r does not have to be small.

Used 3 cells because the total resistance was comparatively high.

Parallax bothered some in getting the balance.

TABLE XXII

Galv. 2717 C; coil 330; 3 dry cells.

No.	P	Q'	Q	R	r		S'	a mm.	C	L
(1)	40	36.1	40.02	200	99	99	200	818	2.85	.00559
(2)	40	36.1	40.02	200	167	168	200	816	1.0	.00561
(3)	40	36.1	40.02	300	231	233	300	800	.8	.00564
(4)	30	26.1	30.01	300	264	266	300	792	.8	.00562
(5)	30	26.1	30.01	250	215	215	250	793	1.0	.00555

None discarded. No difficulty except the slight parallax.

TABLE XXIII

Galv. 2183; coil 330; 3 dry cells.

No.	P	Q'	Q	R	r		S'	a mm.	C	L
(1)	30	26.1	30.02	250	227	227	250	790	.9	.00557
(2)	30	26.1	30.01	300	282	282	300	786	.7	.00557
(3)	30	26.1	30.01	300	264	264	300	785	.8	.00558
(4)	40	36.1	40.02	200	167	167	200	807	1.0	.00558
(5)	40	36.1	40.02	200	113	113	200	808	2.0	.00557

None discarded. Note agreement in values of r.

It should be noticed that in these last two tables the galvanometers are different, but all other conditions are exactly the same. Hence it affords a comparison of the two galvanometers. The agreement of results for the Nalder is seen to be very much better than for the Type-H. But this does not

mean that 2183 is more sensitive than 2717 C, for it is not. There are two reasons for it:- (1), The scale distance for the Nalder is about twice as great as for the Type-H. (2), With a spot of light on the scale directly before the eye, one can detect a very small kick with more certainty than when looking thru ~~the~~ a telescope.

,TABLE XXIV

Galv. 2183; coil 660; 3 dry cells.

No.	P	Q'	Q	R	r		S'	a mm.	C	L
(1)	30	22.1	30.03	400	400	400	400	274	2.0	.0240
(2)	30	22.1	30.03	400	326.5	326.5	400	281	3.0	.0240
(3)	30	22.1	30.04	350	306	306	350	310	3.0	.0241
(4)	40	32.1	40.05	300	268.5	268.5	300	354	2.5	.0241
(5)	40	32.1	40.06	300	286.5	286.5	300	360	2.2	.0241

None discarded. Note the agreement in corresponding values of r. The sensitiveness of the method was not lost by making r = 400 ohms.

Summary of Conclusions.- 1. A sliding contact between the reistance boxes, r and r', is not necessary.

2. As in Anderson's method, the balance should be obtained first by increasing r, then by decreasing r, and the mean taken.

3. The value of r should be nearly equal to that of R.

4. R should be large and C small, so that r will be comparatively large. Thus, the percent of error due to any mistake in the value of r is decreased. Since the r in this method is not in the galvanometer circuit, there is no ob-

jection to increasing its value.

5. Q should be computed by the formula Q = PS/R, rather than built up from the resistance of the coil. (See page 14).

6. With coils of small resistance a resistance in series should be used.

7. The Ayrton-Mather and Nalder galvanometers gave better results than the Type-H. The Type-H is not so well adapted for zero methods. (See page 23).

8. The greater sensitiveness of the Ayrton-Mather is of ne advantage unless it will come completely to rest. (See 5, page 18).

9. When using a rather high total resistance or when using a coil of small inductance, three cells proved more satisfactory than two; but, in general, the extra cell is not desirable because of the added trouble to keep the bridge balanced for steady currents.

RUSSELL'S MODIFICATION OF MAXWELL'S METHOD[1]

Theory and Manipulation.- The theory is the same as for Maxwell's method and, in fact, the arrangement of apparatus is also exactly the same. But no attempt is made to balance the bridge for varying currents. Suppose the steady-current balance is obtained. With a certain value of the capacity, the deflection in one direction is noted; then the capacity is changed enough to give a throw in the opposite direction, and these values are recorded. The value of C which would reduce the deflection to zero is found by interpolation and substituted in Maxwell's formula: L = QRC.

Data and Discussion.- This notation throughout:- L in

[1] London Electrician, May 4, 1894.

henrys, C in microfarads, a in millimeters, d in millimeters, resistances in ohms.

In making measurements with this method, the greatest difficulty is to get deflections large enough to be read accurately.

TABLE XXV

Galv. 2557 B; coil 132; 3 dry cells.

				C	P	Q'	Q	R	S'	a	L
c	0	.3	.4	.4	20.9	20	20.73	121	120	624	.001003
d	7	1.8	.0								
c	0	.25	.3	.3	24.9	24	24.74	131	130	605	.000971
d	6	1.0	.0								
c	0	.25	.3	.3	30.7	30	30.75	110	110	619	.001014
d	10	1.0	.0								

None discarded. For (3), 5 cells were used, and G was not steady. For (1) and (2) G was steady.

It happened in each case that a value of C was found, without interpolating, which made d = 0. Note in (3) that .05 m.f. change in C makes only 1mm. change in d. Obviously, then, the value of C for balance cannot be found accurately.

The inductance is too small for this method.

TABLE XXVI

Galv. 2557 C; coil 1527; 2 dry cells.

				C	P	Q'	Q	R	S'	a	L
c	2.09	3.98	0	3.59	13	10	13.13	119	120	430	.00561
d	+7	-1.8	+17								
c	0	2.09	6.07	3.64	13	10	13.13	120	121	428	.00573
d	+17	+7	-11								
c	0	1.0	2.09	2.02	23	20	23.22	120	121	575	.00563
d	14	+7	-.5								
c	0	2.09	3.09	3.21	23	20	23.10	75	75	380	.00556
d	+18	+6	+.8								
c	0	3.09	5.07	4.08	23	20	23.12	60	60	407	.00566
d	+20	+5	-5								

None discarded. The agreement of results is not good, due to the fact that such small deflections can not be read accurately. G was steady.

TABLE XXVII

Galv. 2557 C; coil 1523; 2 dry cells.

				C	P	Q'	Q	R	S'	a	L
c	0	2.09	3.09	2.49	15	10	15.04	120	120	225	.00449
d	+13	+2	-3								
c	0	2.09	3.09	2.89	15	10	15.06	100	100	249	.00435
d	+14	+4	-1								
c	0	2.09		2.09	15	10	15.03	140	140	226	.00440
d	+11	+0									
c	0	2.09	3.09	3.00	18	13	18.08	80	80	304	.00434
d	+15	+5	-.5								
c	0	2.98	3.48	3.23	18	13	18.08	75	75	316	.00438
d	+16	+1	-1								

None discarded. G steady.

TABLE XXVIII

Galv. 2557 C; coil 133; 1 dry cell.

				C	P	Q	R	S'	a	Q'	L
C	0	29.14	32.1	29.95	30.1	29.86	110	110	450	0	.0993
d	+120	+3	-8								

None discarded. Had seven of the larger condensers in parallel. A smaller capacity could have been used, of course, by making R and S' larger. G steady.

TABLE XXVIX

Galv. 2002; coil 1523; 2 dry cells.

				C	P	Q'	Q	R	S'	a	L
C	0	2.09	2.98	2.39	18	13	18.07	100	100	292	.00432
d	+5	+.75	-1.5								
C	0	3.09	5.07	4.41	14	9	14.13	70	70	309	.00436
d	+6.2	+2	-1								

None discarded. So small an L cannot be measured accurately with this galvanometer, for d is too small when C = 0. λ is too large.

TABLE XXX

Galv. 2183; coil 1520; 2 dry cells.

				C	P	Q'	Q	R	S'	a	L
C	0	2.09	3.98	3.533	18	12	18.26	99	100	393	.00639
d	+19	+8	-2.5								
C	0	2.51	3.51	3.557	18	12	18.25	99	100	388	.00643
d	+19	+5.5	+.25								
C	0	1.0	3.91	3.543	18	12	18.26	99	100	365	.00639
d	+19	+14	-2								
C	0	1.0	5.42	2.87	20	14	20.26	109	110	365	.00634
d	+18	+11	-15								

None discarded. The only difficulty is to read correctly such small deflections.

TABLE XXXI

Galv. 2557 D; coil 460; 2 dry cells.

			C	P	Q'	Q	R	S'	a	L
C	.4	3.35	1.33	30	25	30.06	200	200	424	.01040
d	+14	-17								
C	.3	5.90	3.536	30	25	30.10	100	100	490	.01063
d	+26	-19								
C	.5	4.05	3.540	30	25	30.10	100	100	489	.01065
d	+24	-4								
C	.5	6.90	4.341	30	25	30.12	80	80	514	.01046
d	+27	-18								
C	.4	6.90	5.114	20	15	20.07	100	100	442	.01026
d	+29	-11								

None discarded. Values of d for C u O were not recorded. Note that (3) and (5) are off the most and also that the negative

deflections in these two cases are the two smallest. (The correct value of L is about .01045). G steady.

TABLE XXXII

Galv. 2557 D; coil 590; 2 dry cells.

			C	P	Q'	Q	R	S'	a	L
c	1.	12.32	9.249	20	13	20.05	100	100	630	.0185
d	+51	-19								
c	3.05	11.32	9.253	20	13	20.05	100	100	631	.0186
d	+39	-13								
c	1.	5.90	2.818	20	13	20.01	300	300	684	.0185
d	+17	-23								
c	.8	4.05	2.646	20	13	20.01	350	350	684	.0185
d	+15.8	-12								

Discarded one determination which was off by 11% because of some mistake. G steady.

The table shows that the larger L can be measured more accurately with this method.

TABLE XXXIII

Galv. 2557 D; coil 133; 2 dry cells.

			C	P	Q'	Q	R	S'	a	L
c	2.51	6.42	4.242	30	0	29.79	806	800	585	.1018
d	+23	-29								
c	1.	5.42	3.772	30	0	29.81	900	894	510	.1010
d	+37	-22								
c	1.	5.	3.770	30	0	29.81	900	894	400	.1011
d	+18	-8								
c	1.	5.	3.35	30	0	29.80	1000	993	320	.0998
d	+15.5	-11								

None discarded. Note that for (3), and (4), standard condensers

were used. There is a constant error in (1), (2) and (3), of 1% or more. Since (3), however, is no improvement over (1) and (2), the error is not traceable to the capacities. It is, no doubt, some avoidable experimental error. One cell was used for (3) and (4).

TABLE XXXIV

Galv. 2717 C; coil 330; 2 dry cells.

					C	P	Q'	Q	R	S'	a	L
(1)	C	0	.7	2.	1.74	30	26.1	30.10	100	100	473	.00525
	d	+5	+3	-.75								

Made no more than the one determination because d is too small when C = 0, to be read with any degree of accuracy. An L of this value should not be measured with this galvanometer and method.

TABLE XXXV

Galv. 2717 C; coil 660; 3 dry cells.

					C	P	Q'	Q	R	S'	a	L
(1)	C	0	.8	5.07	2.667	30	22.1	30.04	300	300	323	.0240
	d	+10	+7	-9								
(2)	C	0	1.	4.04	2.658	30	22.1	30.04	300	300	335	.0240
	d	+10	+6	-5								
(3)	C	0	1.	6.89	3.945	30	22.1	30.06	200	200	370	.0237
	d	+14	+10	-10								
(4)	C	0	6.89	1.	3.244	30	22.1	30.05	250	250	352	.0244
	d	+11	-13	+8								
(5)	C	0	6.89	1.	3.191	30	22.1	30.05	250	250	352	.0240
	d	+11	-13.5	+8								

None discarded. (4) and (5) illustrate well the source of error

due to using small deflections. In these two determinations all the conditions are exactly the same, (5) being a repitition of (4). However, in (5) the negative deflection was read 1/2 mm. more than in (4) andthe results show that it made a difference of over 1% in L. The agreement of results in this table and others is as good as it is, only because the greatest possible care was taken in reading the deflections.

Summary of Conclusions.- 1. As this is a deflection method, the greatest source of error , naturally, was found to be in the reading of the deflections.

2. In all the data recorded, the deflections were obtained by simply breaking the battery circuit after the bridge was balanced for steady currents; and the results are good for the coils of comparatively large inductance. However, forthe coils having an inductance less than about .01 henry, the agreement of results was not very satisfactory because the deflections were too small to be read accurately. Hence, to measure the smaller coils, some means of increasing the galvanometer throws ought to be adopted.

There are two ways of doing this: (1) by reversing the current with a Pohl commutator, instead of merely braking the battery circuit; and (2) by using the pendulum apparatus to open the galvanometer circuit immediately after the battery circuit is broken, and thereby avoid most of the damping. But there is an objection to the first means. The successive time intervals taken to throw over the rocker of the commutator will not be exactly equal, and this will introduce a source of

error in the deflections. As to the second, the pendulum apparatus does not have this objection and , moreover, it was found by trial to give deflections as large as are got by reversing the current with the galvanometer key closed. (See the discussion of the use of the pendulum apparatus in Lord Rayleigh's method,page 35). Breaking the circuits in this way is a very easy matter, and it doubtless would greatly improve the results possible with Russel's method.

(3). The sensitiveness of the galvanometers made more difference than for the two former methods, the Ayrton-Mather giving much the best results. The Type-H and Nalder 2183 did very well, but Nalder 2002 was not satisfactory at all, because of its high logarithmic decrement.

LORD RAYLEIGH'S METHOD[1]

Theory and Manipulation.- In this method, which was due originally[2] to Maxwell, a capacity is not used. The inductance coil L, Fig. 4, is connected with three non-inductive resistances in the Wheatstone's bridge. These resistances are first adjusted so as to obtain the steady-current balance.

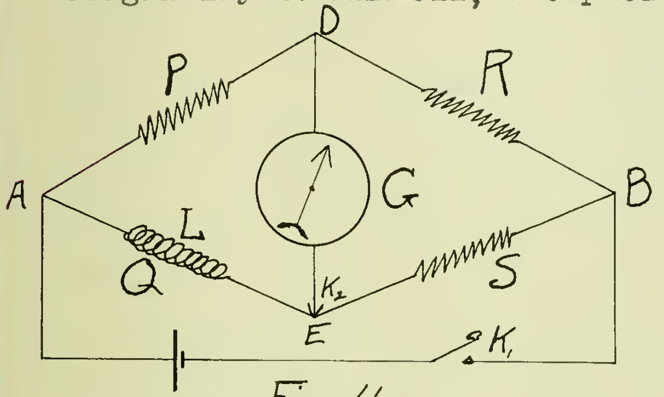

Fig 4.

Then if K_1 is closed after K_2, a quantity of electricity will be sent thru the galvanometer, due to the ~~influenced~~ induced E.M.F. in

1. Pro. Roy. Soc., Vol. XXXII. (1881)., p. 116; also Phil. Trans. (1882)., part 2.
2. Phil. Trans. R.S., Vol. CLV, 1865; or Clerk Maxwell's "Collected Papers", Vol. I., p. 547.

the coil. This quantity can be calculated as follows:[1] An electromotive force of value, e, in the branch AE, will send thru G a current proportional to e; say Ke.

But the induced E.M.F. in L is

$$-L\,di/dt$$

where i is the instantaneous value ofthe current in the coil. Consequently, the current c produced in the galvanometer because of this E.M.F. is

$$C = -KL\,di/dt$$

Let Q represent the total quantity which flows thru the galvanometer. Then

$$Q = \int c\,dt = -KL\int di/dt \times dt = -KL\int_0^I di = -KLI,$$

where I is the final steady value of i.

From the theory of the ballistic galvanometer, this quantity can also be expressed by the equation

$$Q = (HT/\pi G)(\sin\alpha/2)(1 + \lambda/2).$$

Therefore $KLI = (HT/\pi G)(\sin\alpha/2)(1 + \lambda/2)$,

and $L = (HT/\pi GKI)(\sin\alpha/2)(1 + \lambda/2)$ ....(16).

The resistance Q is now changed by a very small amount, Δr, and so the balance for steady currents destroyed. This will cause a current c' to flow thru the galvanometer and produce a steady deflection, θ. But because of the smallness of c', we may assume that the value of the current in the branch AE is still I. However, the introduction of the resistance Δr into this branch has the same effect as if an E.M.F. of value ΔrI were introduced, and so the current thru the galvanometer is

1. J. J. Thompson, Elect. and Mag., p. 434.

seen to be

$$c' = K\Delta rI.$$

But $$C' = (H/G)(\tan\theta); \text{ and, therefore,}$$

$$K\Delta rI = (H/G)(\tan\theta,$$

or $$KI = H\tan\theta/G\Delta r \qquad (17).$$

Substituting this value of KI in (16), we have

$$L = \frac{T\Delta r\sin\alpha/2(1 + \lambda/2)}{\pi\tan\theta}.$$

In this formula, T is the complete period of the needle and α and θ are the angular deflections. L is expressed in henrys if r is in ohms.

A Difficulty.- Because of the fact that the galvanometer key is kept closed while the induction current is flowing, it is obvious that the value of λ, used in the formula, should be determined with the galvanometer on closed circuit. It is impossible, however, in the case of the d'Arsonval type of galvanometer, to get a satisfactory value of λ on closed circuit unless the instrument has a very high resistance- 2000 ohms, or more. To avoid this difficulty, special keys were used in both the galvanometer and the battery circuit, and they were tripped by means of a pendulum apparatus.

The pendulum apparatus was one of those used in the electrical laboratory for condenser experiments. A sketch of it is shown in Fig. 5. The bob of the pendulum swings between two parallel, circular arcs of wood upon which the keys are clamped. The keys are kept closed by means of small lever de-

tents which project up vertically. When, however, one of these

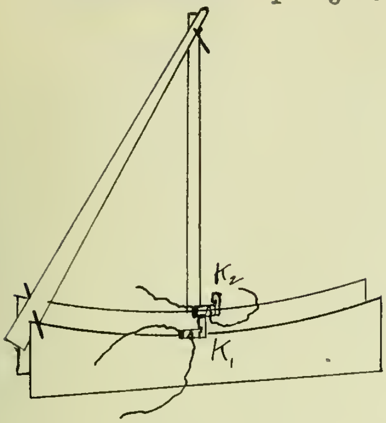

Fig. 5.

levers is thrown over by the cross bar of the pendulum, the spring contact is released and the corresponding circuit opened instantly. Since the positions of the two keys are adjustable, the time interval between the opening of the two circuits by this device can be made as small as we please. The pendulum is about a meter long.

Figure 6 shows the connections. K_1 and K_2 were used to break the battery and galvanometer circuits, respectively. K_1 was clamped and kept stationary. Starting with K_2 several centimeters behind, it was gradually moved up toward K_1 and observations of the throw were made after each change in its position, till the deflection reached a maximum. This position was about a centimeter behind K_1, and it was found

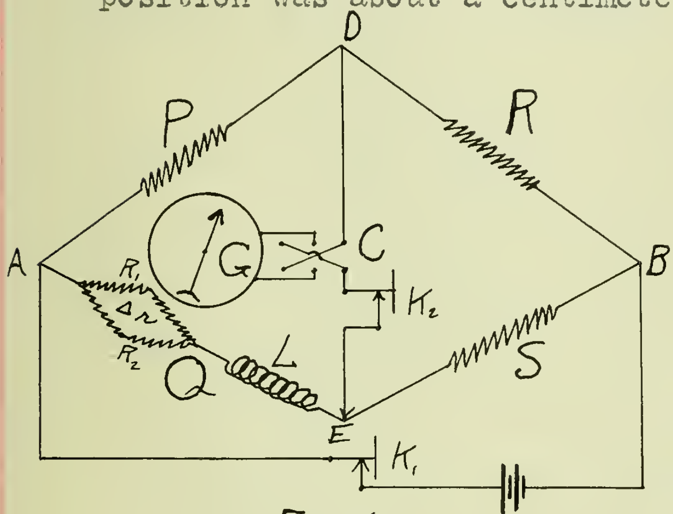

Fig. 6.

that the throw did not decrease any until this distance was reduced to a millimeter. This process was repeated for different coils (.001 henry to .1 henry) with four galvanometers, and practically no difference could be observed in the range of positions of K_2 for which the throw was a maximum. This indicates that K_2 does not have to be accurately placed. It can be set by guess at a distance of about 5 mm. behind the battery key and will not need to be moved for any of the coils ordinarily measured in laboratory

experiments.

Advantage of using the Pendulum Apparatus.- There are three advantages in using the pendulum apparatus.

(1). The value of λ, determined with the galvanometer on open circuit, can be used. As has already been said, it is necessary to use this for most d'Arsonval galvanometers because, on closed circuit, the spot of light will swing past the zero only one or two times, making it impossible to determine the logarithmic decrement.

(2). Since the damping is so slight when the galvanometer circuit is opened immediately after the battery circuit, the ballistic throw is very much increased and hence can be read more accurately. Of course, the gain in this way depends upon the resistance of the galvanometer; but it was found by actual trial that the throw is generally doubled. And so we have deflections as large as would be got by reversing the current with K_2 kept closed.

(3). The value of λ for a galvanometer on closed circuit varies with the resistance in the circuit. Hence, in case one is using a type of galvanometer for which λ can be readily determined with the key closed, it is then necessary to make a number of determinations for different values of circuit resistance and plot a curve, before working values of the decrement can be obtained. By using the pendulum apparatus this trouble and time is saved, for λ on open circuit is constant.

Referring again to Fig. 6, R_1 and R_2 are the two sides of a resistance box connected in parallel for the purpose of getting the small values of $\underline{\Delta r}$. The commutator, C, in the galv-

anometer circuit was introduced in order to have the steady deflection in the same direction as the ballistic throw. This was found to improve results, the reason being, of course, that most galvanometer suspensions have a slightly different resistance to torsion in different directions. The use of the commutator for this purpose, however, was soon discarded as it was discovered that the same end could be accomplished by making Δr negative.

Care necessary.- It was found out immediately that in order to make accurate measurements with Lord Rayleigh's method, the utmost care is necessary in getting the deflections. The chief difficulty lies in the fact that the steady balance changes continually, due to variation in the temperature of the coil and connecting wires, making it necessary to take several readings in as short a time as possible. Hence, the coils were always wrapped with thick felt; but even then the balance would generally be off, by the time one reading was made of d_α and d_θ.

It was found best to make the determinations of d_α and d_θ as follows: The amount of resistance to be plugged in at R_2 in order to make d_α approximately equal to d_θ was decided by preliminary trial. Then, having balanced the bridge carefully by means of the sliding key at E, the pendulum was released and the reading of d_α put down. K_1 and K_2 were now set up and the plugs put in at R_2 as quickly as possible, and as soon as d_θ was read, the plugs at R_2 were removed again so that the balance could be tested. If, as generally happens, the galvanometer then showed a permanent deflection of a millimeter or

more, indicating the bridge balance was off by that time, a suitable correction in the value of d_θ had to be made. But this correction in most cases was only one-half of the permanent deflection referred to, because the interval up to the time when d_θ was read was generally about one-half the whole interval from the time the original balance was made until the balance was tested. Having thus determined one d_α and the corresponding d_θ, the bridge was re-balanced and the whole process repeated for another d_α and d_θ; and so on, as fast as possible, till four or five pairs of values were obtained. The mean of these values, in each case, was the one recorded in the data.

These independent determinations often disagreed by as much as one or two millimeters, owing to variations in the battery, when dry cells were used; but for storage cells the agreement was much better. Hence, a storage battery should be used.

Another source of error in getting the deflections should be mentioned. Some galvanometers gradually change their zero when a current is sent thru them in the same direction a number of times. Consequently, the position of the zero was observed at each time the bridge was re-balanced. A deflection due to the zero being off affects the value of both d_α and d_θ, and should not be confused with the case where the deflection is caused by an imperfect balance and, under the conditions discussed above, necessitates a correction in d_θ only. If the zero is dragged along fast enough to be noticed after a single pair of readings are taken, instead of correcting for it, the difficulty can be avoided by using a commutator in either the galvanometer or battery circuit; so that alternate pairs of corresponding

deflections are in opposite directions.

After making a number of determinations it was found to be of advantage to have about the same current flowing thru the opposite sides of the bridge. Hence, the galvanometer and battery were interchanged as shown in Fig. 7. By keeping P = R, the values of the current in the right and left sides of the bridge are always equal. This is the final arrangement of apparatus which gave the best results.

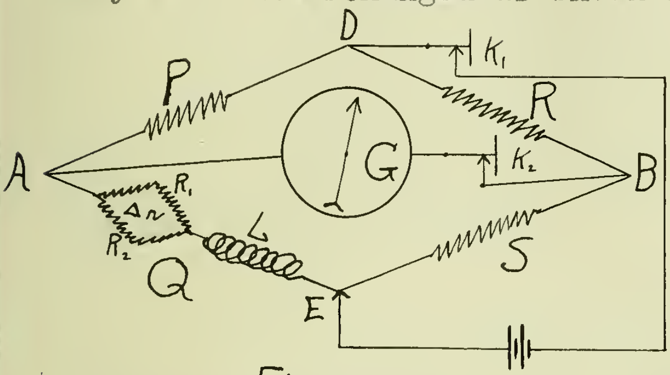

Fig. 7.

Data and Discussion.- The value of λ for each galvanometer was got by taking the mean of five independent determinations. The period was obtained by counting five times over the interval of 30 vibrations. (For values of λ and T, see page 3).

TABLE XXXVI

Galv. 2183; coil 660; 1 storage cell.

Apparatus as in Fig. 6

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	L
(1)	72	70	1°51.3'	.0162	1°48'	.0314	.0318	.0247
(2)	102	97	2°38	.0230	2°29'	.0434	.0318	.0251
(3)	196	112	5°0'	.1009	2°52'	.0501	.0190	.0246
(4)	136	98	3°30'	.0304	2°31.3	.0440	.0245	.0251
(5)	187	228	4°30'	.0393	5°48'	.1016	.0418	.0246
(6)	69.5	84.5	1°47.5′	.0156	2°10.5	.0380	.0418	.0255

None discarded. The agreement is very poor.

TABLE XXXVII

Glav. 2557 C; coil 660; apparatus as in Fig. 6.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	dry cells	L
(1)	98	137	2°36'	.0227	3°37'	.0632	.0418	3	.0237
(2)	97	78	2°34'	.0224	2°04'	.0361	.0245	3	.0240
(3)	67	72	1°46'	.0154	1°54'	.0332	.0326	2	.0263
(4)	94	102	2°29'	.0218	2°42'	.0472	.0326	1	.0238

None discarded. Agreement still worse, due no doubt to variations in the dry battery.

The above two groups of data were not taken till after a number of preliminary determinations had been made. So they do not represent the first attempt with the method.

TABLE XXXVIII

Galv. 2557 C; coil 660; app. as in Fig. 7; dry cells.

	d_α	d_θ	α	$\sin\frac{\alpha}{2}$	θ	$\tan\theta$	Δr	P = R	no. of cells	L
(1)	94	101	2°29!3	.0218	2°40'	.0466	.0326	17	1	.0242
(2)	124	114	3°16!5	.0285	3°05'	.0526	.0280	27	2	.0241
(3)	124	116	3°16!5	.0285	3°04'	.0536	.0280	37	2	.0236
(4)	128	118	3°23'	.0295	3°07'	.0544	.0280	37	3	.0241
(5)	109	101	2°53'	.0250	2°40'	.0466	.0280	60	1	.0238

None discarded. Q = S = 17, approx. These results are seen to be much better. No (3) is a repetition of (2) under exactly the same conditions. Note that the difference of 2 mm. in d_θ changed L by 2 percent. This error was due to a variation in the battery.

TABLE XXXIX

Galv. 2557 C; coil 660; app. as in Fig. 7; storage cells.

	d_α	d_θ	α	$\sin \alpha/2$	θ	$\tan \theta$	Δr	No. Cells	P=R	$Q_1=S_1$	L
(1)	80	74	2°07'	.0185	1°58'	.0342	.0280	1	60	27	.0240
(2)	86	94	2°16'.5	.0199	2°29'	.0434	.0333	1	30	27	.0242
(3)	126.5	101	3°20'.3	.0291	2°40'	.0466	.0245	1	30	17	.0243
(4)	163.8	1312	4°19'	.0376	3°28'	.0606	.0245	2	80	17	.0241
(5)	144	1155	3°48'	.0332	3°03'	.0533	.0245	2	100	17	.0242

None discarded. The agreement is still better. Hence, storage cells should be used.

Q_1 and S_1 are the resistances in the Q and S branches of the bridge, respectively, <u>not</u> including the resistance of the corresponding parts of the slide wire. It was not necessary to compute the exact values of Q and S, since they are not used in the formula.

TABLE XL

Galv. 2557 C; coil 790; app. as in Fig. 7; storage cells.

	d_α	d_θ	α	$\sin \alpha/2$	θ	$\tan \theta$	Δr	P=R	$Q_1=S_1$	No. Cells	L
(1)	189	194	4°58'	.0433	5°5'.5	.0891	.0419	100	19	2	.0323
(2)	157	160.5	4°08'	.0361	4°12'	.0734	.0419	130	19	2	.0327
(3)	137.5	120.8	3°38'	.0317	3°11'	.0556	.0361	130	29	2	.0326
(4)	102	90	2°42'	.0236	2°23'	.0416	.0361	60	29	1	.0325
(5)	113	100	2°59'	.0262	2°39'	.0463	.0361	40	29	1	.0324

None discarded.

TABLE XLI

Galv. 2557 C; coil 790; app. as in Fig. 7; 1 storage cell.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	P	Q_1	R	S_1	L
(1)	115.5	101.5	3°1!5	.0265	2°41'	.0469	.0361	40	29	40	29	.0324
(2)	108	112.2	2°51'	.0250	2°58'	.0518	.0428	29	29	50	50	.0328
(3)	98.2	102.2	2°36'	.0227	2°42'	.0472	-.0428	29	29	70	70	.0327
(4)	92.6	97.6	2°27'	.0214	2°35'	.0451	-.0428	29	29	80	80	.0322
(5)	82.6	87.0	2°11'	.0191	2°18'	.0402	-.0428	29	29	100	100	.0323

None diacarded. In (4) and (5), Δr was made negative and so the commutator was not used.

TABLE XLII

Galv. 2557 C; coil 790; app. as in Fig. 6; 2 dry cells.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	P	Q_1	R	S_1	L
(1)	114.5	120.5	3°1!5	.0265	3°11'	.0556	-.0428	29	29	100	100	.0324
(2)	104.5	71.6	2°46'	.0241	1°54'	.0331	-.0280	29	29	120	120	.0323
(3)	91.5	97	2°25'	.0212	2°34'	.0448	-.0426	29	29	150	150	.0320
(4)	92	96	2°26'	.0213	2°32'	.0442	-.0426	29	29	150	150	.0326
(5)	100	103	2°39'	.0232	2°43!5	.0476	-.0418	191	19	150	152	.0323

None discarded. No's (3) and (4) have exactly the same resistances, showing again how variations in the E.M.F. affect the result. Dry cells are not as satisfactory as the storage cells.

TABLE XLIII

Galv. 2557 C; coil 133; app. as in Fig. 6.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	$P=Q_1$	$R=S_1$	L
(1)	232	119	6°04'	.0529	3°8.5'	.0549	-.0649	40	160	.0992
(2)	179	92	4°42.5'	.0411	2°26'	.0425	-.0649	40	40	.0996
(3)	263	135.2	6°50.5'	.0597	3°34'	.0623	-.0649	40	40	.0987
(4)	263	134.2	6°50.5'	.0597	3°32.3'	.0618	+.0649	40	40	.0995
(5)	240	123	6°16'	.0547	3°15'	.0568	+.0649	40	270	.0992

None discarded. 2 dry cells for (1); 1 dry cell for (2); 1 storage cell for (3) and (4); 2 storage cells for (5). The determination of d_α and d_θ is much easier with the storage cells. In (4) and (5), where Δr is plus, the commutator was used to throw the deflections in the same direction.

TABLE XLIV

Galv. 2557 C; coil 133; app. as in Fig. 7; storage cells.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	P=R	$Q_1=S_1$	No. Cells	L
(1)	238	122	6°13'	.0542	3°13.3'	.0563	+.0649	270	40	2	.0996
(2)	240.3	122	6°16.3'	.0547	3°13.3'	.0563	+.0649	270	40	2	.1000
(3)	219	112	5°44'	.0500	2°57.7'	.0517	-.0649	300	40	2	.0996
(4)	207.5	159.4	5°26'	.0474	4°12'	.0734	-.0971	110	40	1	.0995
(5)	176.2	135	4°38'	.0404	3°34'	.0623	-.0971	150	40	1	.0999

None discarded. (2) is a repitition of (1), after the galvanometer was re-leveled. One cell does just as well as two.

The data already given show that a negative Δr gives results just as consistent as when Δr is positive. Hence, from this time on, Δr was always made negative and the commutator in the circuit was no longer used.

Galv. 2002; coils 133 and 590.

This galvanometer, it was found, can not be used with this method because its period is too slow. It was impossible to get a value for d_θ. When the resistance Δr was plugged out to give the steady deflection, the spot of light would move out slowly and with a gradually decreasing rate, and generally would not stop at all. When using the smaller coil (590) it sometimes came almost to rest, but the time interval required was so long that the bridge-balance was off 5-12 mm. by the time d_θ could be read.

It might seem that the trouble was due to rapid change in the room temperature at that time; but such was not the case. The coil was packed with thick felt. Moreover, with a thermometer graduated to tenths of degrees, temperature readings of the coil were taken every ten minutes, and it was found that the rate of change of temperature of the coil was only .5° (5 tenths) for 30 minutes.

Furthermore, the difficulty was not due to using a current too high. The current used for coil 133 was considerably less than was used in several instances with the same coil and galv. 2557 C. And the current for coil 590 was low also,- merely enough to give a throw of 30 mm. One storage cell was used.

Hence, it is seen that a galvanometer with a long period cannot be used with this method.

TABLE XLV

Galv. 2717 A; coil 590; app. as in Fig. 7.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	$P=R$	$Q_1=S_1$	No. cells	L
(1)	57	79	3°12'	.0279	4°25.3'	.0773	.0280	80	26	2	.0185
(2)	61	84.5	3°25!3	.0299	4°43.7'	.0827	.0280	70	26	2	.0185
(3)	65	79	3°38!7	.0318	4°25'	.0773	.0245	90	16	2	.0184
(4)	74.5	91	4°11'	.0365	5°05'	.0890	.0245	40	16	1	.0184
(5)	62	75.6	3°28!7	.0304	4°14'	.0740	.0245	60	16	1	.0184

None discarded. Dry cells for (1)-(3), storage cells for (4) and (5). The Type-H works nicely for this method.

TABLE XLVI

Galv. 2717 A; coil 790 app. as in Fig. 7; storage cells.

	d_α	d_θ	α	$\sin\alpha/2$	θ	$\tan\theta$	Δr	$P=R$	$Q_1=S_1$	No. cells	L
(1)	117	138	6°34'	.0573	7°41.7'	.1351	.0418	40	19.4	1	.0324
(2)	99.3	117.5	5°34.7'	.0487	6°33.7'	.1150	.0418	60	19.4	1	.0324
(3)	85.7	101.6	4°48.7'	.0420	5°40.7'	.0994	.0418	80	19.4	1	.0323
(4)	82	66.3	4°35.8'	.0401	3°43'	.0650	.0280	65	29.4	1	.0316
(5)	87	70	4°52.6'	.0426	3°55.4'	.0686	.0280	170	29.4	2	.0318
(6)	115	138.9	6°28'	.0564	7°44'	.1359	.0426	110	29.4	2	.0323
(7)	102	122	5°43'	.0499	6°50.4'	.1199	.0426	140	29.4	2	.0324
(8)	112.2	133.5	6°17.4'	.0549	7°29'	.1314	.0426	120	29.4	2	.0325
(9)	102	89.5	6°17!4'	.0549	5°01'	.0878	.0280	120	29.4	2	.0320
(10)	102	81	5°43'	.0499	4°32.5'	.0794	.0280	140	29.4	2	.0322
(11)	101	100.3	5°40'	.0494	5°37.4'	.0984	.0351	140	29.4	2	.0322
(12)	110.3	98	6°11'	.0540	5°29.6'	.0962	.0312	120	29.4	2	.0320
(13)	92	91.3	5°9.5'	.0451	5°7'	.0895	.0351	160	29.4	2	.0324

One discarded in which there was evidently a mistake in recording some value. Note that all the resistances in (8) and (9)

are exactly the same, excepting Δr. Also, the resistances in (7) and (10) are the same excepting Δr. Thus, by making d_θ less than d_α, L was made too small. Note that in (4) and (5), where L is again too low, d_θ is less than d_α.

TABLE XLVII

Galv. 2717 A; coil 133; app. as in Fig. 7; storage cells.

	d_α	d_θ	α	$\sin \alpha/2$	θ	$\tan \theta$	Δr	$P=R$	$Q_1 = S_1$	No cells	L
(1)	175	173.6	9°44.3'	.0848	9°39'	.1699	.1099	270	39.5	2	.1003
(2)	122	122.2	6°50.4'	.0596	6°51'	.1201	.1099	170	39.5	1	.0998
(3)	113	113.6	6°20'	.0552	6°22'	.1116	.1099	190	39.5	1	.0994
(4)	121.7	92.1	6°49.4'	.0595	5.10'	.0904	.0823	170	39.5	1	.0991
(5)	121.6	151.6	6°49'	.0595	8°30'	.1495	.1372	170	39.5	1	.0999

None discarded. Here again, the lowest value of L is in (4) where d_θ is less than d_α. Note that (2), (4) and (5) were to test the effect of making d_θ equal to, less than, or greater than d_α.

From the last two tables especially, and also from previous tables, it is evident that the ratio of d_α to d_θ makes a difference in L. When d_θ is less than d_α, L is too small. However, when d_θ is greater than d_α, the data does not indicate a corresponding error in the opposite direction. As to the reason for this, I am not certain. It probably would not prove to be always true. Time was not left to investigate this point enough to justify a general conclusion. It will probably be found that d_α and d_θ ought to be equal, or nearly so, for the best results.

Summary of Conclusions.- 1. The apparatus should be connected as in figure 7.

2. The pendulum apparatus should be used for breaking the circuits.

3. The utmost care is necessary in obtaining the deflections.

4. Δr should be negative in order to throw the two deflections in the same direction. By thus putting the combined resistance R_1 and R_2 in the Q-branch of the bridge, it serves also as a series resistance with the coil.

5. If the zero is gradually dragged off, a commutator should be used. (See page 39).

6. A galvanometer having a slow period (more than 6-8 sec.) cannot be used.

7. The ratio of d_∞ to d_θ makes a difference in the value of L which will be obtained.

8. Coils of comparatively large inductance should be used for this method (.01 henry, or more).

9. No correction is necessary for the fact that the value of the current thru the coil is slightly changed when Δr is introduced.

10. The coil must be protected from rapid changes in temperature.

RÉSUMÉ

In this investigation four methods were studied and compared:- Anderson's, Rimington's, Russell's, and Lord Rayleigh's. Only the d'Arsonval type of galvanometer was used. The zero methods gave more consistent results than the deflection methods. Of the zero methods, Anderson's was somewhat the easier to manipulate, but the two gave equally good results. As to the deflection methods, Russell's was very much easier to manipulate and with ordinary care gave the better results; but when the utmost

care was taken, Lord Rayleigh's method proved as good as Russell's. The Ayrton-Mather galvanometer gave much the best results for the zero methods, and the Nalder was better than the Type-H. (See description of galvanometers, page 3). For the deflection methods, the Type-H was just as good, or even better than the other two kinds of galvanometers. A number of conclusions, in regard to the best working conditions for the different methods, were arrived at and grouped together in four summaries. (See pages 17, 24, 32, 47).